PERGAMON

Chaos, Solitons and Fractals 15 (2003) 611–618

CHAOS
SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

On the mass spectrum of the elementary particles of the standard model using El Naschie's golden field theory

L. Marek-Crnjac *

Faculty of Mechanical Engineering, University of Maribor, Smetanova ulica 17, SI-2000 Maribor, Slovenia

Accepted 30 May 2002

Abstract

In this note we will give an expression of masses of the elementary particles of the standard model in terms of the golden mean. This is the value $\phi = (\sqrt{5}-1)/2$, which corresponds to the Hausdorff dimension of a single random Cantor set. El Naschies transfinite field theory which we will call the golden field theory is based on this fact and is the basis of the present paper.

1. Introduction

In [1,2], the masses of quarks were expressed in terms of ϕ, of the coupling constants $\overline{\alpha}_i$ and of other fundamental particle's mass. In the present work we will give expressions for masses of elementary particles in terms of ϕ and $1/\phi$, where $\phi = (\sqrt{5}-1)/2$ is the golden mean.

The Hausdorff dimension of the zero set $d_c^{(0)}$ is equal to the golden mean value ϕ, if the expectation value of the dimension n and the expectation value of the Hausdorff dimension in the Cantorian space $\mathscr{E}^{(\infty)}$ are equal. This is the condition of space filling:

$$\sim \langle n \rangle = \langle d_c \rangle, \text{ where } \sim \langle n \rangle = \frac{1 + d_c^{(0)}}{1 - d_c^{(0)}}, \ \langle d_c \rangle = \frac{1}{d_c^{(0)}(1 - d_c^{(0)})}$$

If $d_c^{(0)} = \phi = (\sqrt{5}-1)/2$, then

$$d_c^{(n)} = \frac{1}{\left(d_c^{(0)}\right)^{n-1}} = \frac{1}{\phi^{n-1}},$$

where $d_c^{(n)}$ is the Hausdorff dimension of n-dimensional sets $S_c^{(n)}$ of $\mathscr{E}^{(\infty)}$, as shown in [3,4].

* Tel.: +386-2-220-7500; fax: +386-2-220-7990.

0960-0779/03/$ - see front matter
PII: S0960-0779(02)00174-1

Table 1
Mass of subatomic particles, resonance and Gauge bosons following [1,2]

Particle	Theoretical mass in terms of ϕ and $1/\phi$	Theoretical mass in terms of $\overline{\alpha}_0 = (20)(1/\phi)^4 = (20)(7-\phi^4) = 137.082$	The nearest value of the experimental value in powers of ϕ and $1/\phi$	Experimental value
e (Electron)	$\frac{\sqrt{\overline{\alpha}_{gs}}}{10} = \frac{\sqrt{\left(\frac{1}{\phi}\right)^2(10)}}{10} = \left(\frac{\sqrt{10}}{10}\right)\left(\frac{1}{\phi}\right) = \frac{\sqrt{10}}{10}(1+\phi) = 0.51166$ MeV	$\sqrt{\overline{\alpha}_0} = \left(\frac{\sqrt{2}}{10}\right)\phi = 0.51166$ MeV or better $\frac{a_0+2.5}{T_0} = 0.511099$	$\frac{(20)(1/\phi)^4+\phi}{(40)(1/\phi)^4-1} = 0.511414529$ MeV	0.511 MeV
zn (Neutron)	$\frac{\overline{\alpha}_0^2}{20} = \frac{\left((20)\left(\frac{1}{\phi}\right)^4\right)^2}{20} = (20)\left(\frac{1}{\phi}\right)^8 = (20)(47-\phi^8) = 939.574249$ MeV	$\frac{1}{20}\overline{\alpha}_0^2 = 939.574249$ MeV	$(20)\left(\frac{1}{\phi}\right)^8 = (20)(47-\phi^8)$ $= 939.574249$ MeV	939.563 MeV
P (Proton)	$\frac{(\overline{\alpha}_0-k_0)^2}{20} = \frac{\left((20)\left(\frac{1}{\phi}\right)^4-\left(\left(\frac{1}{\phi}\right)^5-11\right)\left(2-\frac{1}{\phi}\right)\right)^2}{20} = \frac{((20)(7-\phi^4)-\phi^5(1-\phi^5))^2}{20}$ $= 938.45$ MeV, where $k_0 = \phi^5(1-\phi^5)$	$\frac{(\overline{\alpha}_0-k_0)^2}{20} = 938.45$ MeV	$\approx (580)\left(\frac{1}{\phi}\right) = (580)(1+\phi)$ $= 938.459$ MeV	938.27231 MeV
$\pi\pm$ (π meson)	$\overline{\alpha}_0 + \frac{5}{2} = \overline{\alpha}_0 + \frac{\overline{\alpha}_0}{8}(\phi^4) = \frac{\overline{\alpha}_0}{8}(8+\phi^4) = \frac{5}{2}(7-\phi^4)(8+\phi^4)$ $= \frac{5}{2}\left(\frac{1}{\phi}\right)^4(8+\phi^4) = 139.5820393$ MeV	$\frac{\overline{\alpha}_0}{8}(8+\phi^4) = 139.5820393$ MeV	$\approx (33)\left(\frac{1}{\phi}\right)^3 = (33)(4+\phi^3)$ $= 139.79$ MeV	139.57 MeV
π^0	$\overline{\alpha}_0 - \frac{5}{2} = \frac{5}{2}(7-\phi^4)(8-\phi^4) = \frac{5}{2}\left(\frac{1}{\phi}\right)^4(8-\phi^4)$ $= 134.5820392$ MeV	$\frac{\overline{\alpha}_0}{8}(8-\phi^4) = 134.5820393$ MeV	$\approx (12)\left(\frac{1}{\phi}\right)^5 = (12)(11+\phi^5)$ $= 133.0820$ MeV	134.98 MeV
$\langle\pi\rangle$	$\frac{1}{2}(m_{\pi^\pm} + m_{\pi^0}) = \overline{\alpha}_0 = (20)\left(\frac{1}{\phi}\right)^4 = (20)(7-\phi^4)$ $= 137.0820$ MeV	$\overline{\alpha}_0 = 137.082$ MeV	$(20)\left(\frac{1}{\phi}\right)^4 = (20)(7-\phi^4)$ $= 137.0820$ MeV	137.275 MeV
$K^\pm$ (Kaon)	$(\mathrm{Dim}\, E_8 \otimes E_8) - 2 = (496-k^2) - 2 = (496-(2\phi^5)^2) - 2$ $= 4\overline{\alpha}_0 - (52+2k+2) = 4\overline{\alpha}_0 - \overline{\alpha}_0\phi^2 - \frac{\alpha_0\phi^4}{10} = \overline{\alpha}_0\left(4-\phi^2-\frac{\phi^4}{10}\right)$ $= (20)(7-\phi^4)\left(4-\phi^2-\frac{\phi^4}{10}\right) = (20)\left(\frac{1}{\phi}\right)^4\left(3+\phi-\frac{\phi^4}{10}\right)$ $= 493.967$ MeV, where $k = \phi^3(1-\phi^3) = 2\phi^5$	$\overline{\alpha}_0\left(4-\phi^2-\frac{\phi^4}{10}\right)$ $= \overline{\alpha}_0\left(3+\phi-\frac{\phi^4}{10}\right) = 493.967$ MeV	$\approx (72)\left(\frac{1}{\phi}\right)^4 = (72)(7-\phi^4)$ $= 493.4953$ MeV	493.646 MeV
K^0	$(\mathrm{Dim}\, E_8 \otimes E_8) + 2 = (496-k^2) + 2 = (20)(7-\phi^4)\left(4-\phi^2+\frac{\phi^4}{10}\right)$ $= (20)\left(\frac{1}{\phi}\right)^4\left(3+\phi+\frac{\phi^4}{10}\right) = 497.967$ MeV	$\overline{\alpha}_0\left(4-\phi^2+\frac{\phi^4}{10}\right) = \overline{\alpha}_0\left(3+\phi+\frac{\phi^4}{10}\right)$ $= 497.967$ MeV	$\approx (190)\left(\frac{1}{\phi}\right)^2 = (190)(3-\phi^2)$ $= 497.42645$ MeV	497.671 MeV

$\langle \mathrm{K}\rangle$	$\frac{1}{2}(\mathrm{K}^{\pm}+\mathrm{K}^{0})=\langle m_{\mathrm{K}}\rangle=(496-k^{2})=(496-(2\phi^{5})^{2})$ $=4\bar{\alpha}_0-(52+2k)=4\bar{\alpha}_0-\bar{\alpha}_0\phi^{2}=\bar{\alpha}_0(4-\phi^{2})$ $=(20)(7-\phi^{4})(4-\phi^{2})=(20)(7-\phi^{4})(3+\phi)$ $=(20)\left(\frac{1}{\phi}\right)^{4}\left(1-\frac{1}{\phi^{2}}\right)=495.967$ MeV	$\bar{\alpha}_0(4-\phi^{2})=\bar{\alpha}_0(3+\phi)=495.967$ MeV	$\approx(117)\left(\frac{1}{\phi}\right)^{3}$ $=(117)(4+\phi^{3})$ $=495.619$ MeV	495.67 MeV
$\Delta(1232)$	$(m_{\pi 0}+4\phi)(9)=\left(\bar{\alpha}_0-\frac{5}{2}+4\phi\right)(9)=\left(\bar{\alpha}_0-\frac{1}{2}\phi^{6}\right)(9)$ $=\left((20)(7-\phi^{4})-\frac{1}{2}+\phi^{6}\right)(9)=1233.48$ MeV	$\left(\bar{\alpha}_0-\frac{1}{2}\phi^{6}\right)(9)=1233.48$ MeV	$\approx(180)\left(\frac{1}{\phi}\right)^{4}$ $=(180)(7-\phi^{4})$ $=1233.48$ MeV	1230–1234 MeV
Ω^{-}	$10[\bar{\alpha}_0+(49)(\phi)]=10[20(7-\phi^{4})+49\phi]$ $=10\left[(20)\left(\frac{1}{\phi}\right)^{4}+(49)\left(\left(\frac{1}{\phi}\right)-1\right)\right]$ $=1673.657047$ MeV	$10[\bar{\alpha}_0+(49)(\phi)]=1673.657$ MeV $\simeq(\bar{\alpha}_0+\sqrt{5})/2=1673.3128$ MeV	$\approx(224)\left(\frac{1}{\phi}\right)^{4}$ $=(224)(7-\phi^{4})$ $=1672.4088$ MeV	1672.43 MeV
Exi^{-}	$10[\bar{\alpha}_0-(8)(\phi)]=10[20(7-\phi^{4})-8\phi]$ $=40[(5)(7-\phi^{4})-2\phi]=1321.377$ MeV	$10[\bar{\alpha}_0-(8)(\phi)]=1321.377$ MeV	$\approx(817)\left(\frac{1}{\phi}\right)=(817)(1+\phi)$ $=1321.9337$ MeV	1321.32 MeV
Exi^{0}	$10[\bar{\alpha}_0-(9)(\phi)]=10[20(7-\phi^{4})-9\phi]$ $=10\left[(20)\left(\frac{1}{\phi}\right)^{4}-(9)\left(\frac{1}{\phi}-1\right)\right]=1315.19733$ MeV	$10[\bar{\alpha}_0-(9)(\phi)]=1315.19733$ MeV	$\approx(28)\left(\frac{1}{\phi}\right)^{8}$ $=(28)(47-\phi^{8})$ $=1315.403$ MeV	1314.9 MeV
τ (tau)	$[10(m_{\mathrm{s}})]-\left[\frac{m_{\mathrm{s}}}{10}\right]=10(10)\left(\frac{1}{\phi}\right)^{6}-\frac{(10)\left(\frac{1}{\phi}\right)^{6}}{10}$ $=99\left(\frac{1}{\phi}\right)^{6}=99(18-\phi^{6})=1776.482919$ MeV	$\bar{\alpha}_0\left(\frac{99}{20}\right)\left(\frac{1}{\phi}\right)^{2}=1776.482919$ MeV $\simeq(137)(13)=1781$ MeV	$99\left(\frac{1}{\phi}\right)^{6}=99(18-\phi^{6})$ $=1776.482919$ MeV	1777 MeV (Donald Parkins)
η	$\frac{[(4)][\bar{\alpha}_{\mathrm{gs}}]^{2}}{20}=\frac{[(40)\left(\frac{1}{\phi}\right)^{2}]^{2}}{20}=(80)\left(\frac{1}{\phi}\right)^{4}=80(7-\phi^{4})$ $=548.3281574$ MeV	$4\bar{\alpha}_0=548.328157$ MeV	$(80)\left(\frac{1}{\phi}\right)^{4}=80(7-\phi^{4})$ $=548.3281574$ MeV	548.8 MeV
Σ^{+}	$\left[\frac{(\bar{\alpha}_{\mathrm{ew}})(m_n)}{100}\right]-(1+\phi)^{3}=\left[\frac{(10)(3)\left(\frac{1}{\phi}\right)^{3}\cdot 20\left(\frac{1}{\phi}\right)^{8}}{100}\right]-\left(\frac{1}{\phi}\right)^{3}$ $=\left[6\left(\frac{1}{\phi}\right)^{3}\left(\frac{1}{\phi}\right)^{8}-\left(\frac{1}{\phi}\right)^{3}\right]=\left(\frac{1}{\phi}\right)^{3}\left[6\left(\frac{1}{\phi}\right)^{8}-1\right]$ $=(4+\phi^{3})[6(47-\phi^{8})]$ $=1189.79$ MeV, where $\bar{\alpha}_{\mathrm{ew}}=(10)(3)\cdot\left(\frac{1}{\phi}\right)^{3}$	$\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^{3}\frac{\bar{\alpha}_0^{2}}{20}-\left(\frac{1}{\phi}\right)$ $=\left(\frac{1}{\phi}\right)\left(\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^{2}\frac{\bar{\alpha}_0^{2}}{20}-1\right)$ $=1189.79$ MeV	$\approx(735)\left(\frac{1}{\phi}\right)=(735)(1+\phi)$ $=1189.2549$ MeV	1189.37 MeV

(*continued on next page*)

Table 1 (*continued*)

Particle	Theoretical mass in terms of ϕ and $1/\phi$	Theoretical mass in terms of $\overline{\alpha}_0 = (20)(1/\phi)^4 = (20) \times (7-\phi^4) = 137.082$	The nearest value of the experimental value in powers of ϕ and $1/\phi$	Experimental value
Σ^0 (sigma)	$\left[\frac{\overline{\alpha}_{ew}(m_n)}{100}\right] + \frac{2}{3}(1+\phi)^3 = \left[\frac{(10)(3)\left(\frac{1}{\phi}\right)^3\cdot 20\left(\frac{1}{\phi}\right)^8}{100}\right] + \frac{2}{3}\left(\frac{1}{\phi}\right)^3$ $= \left(\frac{1}{\phi}\right)^3\left[(6)\left(\frac{1}{\phi}\right)^8 + \frac{2}{3}\right]$ $= \left((4+\phi^3)\left[(6)(47-\phi^8)+\frac{2}{3}\right]\right) = 1196.854$	$\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^3\frac{\overline{\alpha}_0^2}{20} + \frac{2}{3}\left(\frac{1}{\phi}\right) = \left(\frac{1}{\phi}\right)$ $\left(\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^2\frac{\overline{\alpha}_0^2}{20} + \frac{2}{3}\right) = 1196.854$ MeV	$\approx (740)\left(\frac{1}{\phi}\right)$ $= (740)(1+\phi)$ $= 1197.34515$ MeV	1197.43 MeV
Σ^-	$\left[\frac{\overline{\alpha}_{ew}(m_n)}{100}\right] - \frac{1}{3}(1+\phi)^3 = \left(\frac{1}{\phi}\right)^3\left[(6)\left(\frac{1}{\phi}\right)^8 - \frac{1}{3}\right]$ $= \left((4+\phi^3)\left[(6)(47-\phi^8)-\frac{1}{3}\right]\right) = 1192.618$ MeV	$\frac{1}{\phi}\left(\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^2\frac{\overline{\alpha}_0^2}{20} - \frac{1}{3}\right) = 1192.618$ MeV $\cong (\overline{\alpha}_0 - (4+\phi))(9) = 1192.17$ MeV	$\approx (737)\left(\frac{1}{\phi}\right) = (737)$ $(1+\phi) = 1192.49$ MeV	1192.55 MeV
$\langle\Sigma\rangle$	$\left[\frac{\overline{\alpha}_{ew}(m_n)}{100}\right] = (6)\left(\frac{1}{\phi}\right)^{11} = 6(199+\phi^{11}) = 1194.030149$ MeV	$\frac{\overline{\alpha}_0^2}{20}\left(\frac{3}{10}\right)\left(\frac{1}{\phi}\right)^3 = 1194.030149$ MeV $\simeq (\overline{\alpha}_0 - 4.5)(9) = 1192.38$ MeV	$\approx (6)\left(\frac{1}{\phi}\right)^{11} = 6(199+\phi^{11})$ $= 1194.030149$ MeV	1193.28 MeV
μ (meuon)	$m_\mu = \sqrt{(10)^3\left(\frac{1}{\phi}\right)^5} = (10)\left(\frac{1}{\phi}\right)^2\sqrt{10\left(\frac{1}{\phi}\right)}$ $= 10(3-\phi^2)\sqrt{10(1+\phi)} = 105.3098759$ MeV	$\sqrt{\overline{\alpha}_0}\sqrt{(2)\left(\frac{1}{\phi}\right)}(5)$ $= \sqrt{\overline{\alpha}_0}\sqrt{(2)\left(\frac{1}{\phi}\right)}\ (2+\phi^3)^2$ $= 105.309875$ MeV	$\approx (25)\left(\frac{1}{\phi}\right)^3$ $= (25)(4+\phi^3)$ $= 105.6838$ MeV	105.65839 MeV
$\frac{m_\mu}{m_e}$	$\frac{m_\mu}{m_e} = \frac{\sqrt{(10)^3\left(\frac{1}{\phi}\right)^5}}{\sqrt{\overline{\alpha}_{gs}}} = \sqrt{(10)^4\left(\frac{1}{\phi}\right)^3} = (10)^2\sqrt{\left(\frac{1}{\phi}\right)^3}$ $= (10)^2\sqrt{(4+\phi^3)^3} = 205.8171028$ MeV	$(10)\sqrt{\overline{\alpha}_0}\sqrt{(2+\phi^3)\phi}$ $= 205.8171028$ MeV where $\sqrt{5} = (2+\phi^3)$	$\approx (79)\left(\frac{1}{\phi}\right)^2$ $= (79)(3-\phi^2)$ $= 206.82468$ MeV	206.768262 MeV
W	$(\overline{\alpha}_0)(1-\sin^2\theta_w)^2(10)^3 = (20)\left(\frac{1}{\phi}\right)^4(1-\phi^3)^2(10)^3$ $= (10)^3(20)\left(\frac{1}{\phi}\right)^4(4)(\phi)^4 = 80\left(\frac{\phi}{\phi}\right)^4 = 80(7-\phi^4)(\phi^4)$ $= 80$ GeV, where $\sin^2\theta_w = \phi^3$ and $(1-\phi^3) = (2)(\phi)^2$	$4\overline{\alpha}_0(\phi)^4 = 80$ GeV	$\approx (404)\left(\frac{1}{\phi}\right)^{11}$ $= (404)(199\ +\phi^{11})$ $= 80.398$ GeV	80.4 GeV
Z	$\frac{m_w}{\sqrt{1-\phi^3}} = \frac{80}{\sqrt{(2)(\phi)^2}} = \left(\frac{80}{\sqrt{2}}\right)\left(\frac{1}{\phi}\right)$ $= (40\sqrt{2})(1+\phi) = 91.5298244$ GeV	$(2\sqrt{2})\overline{\alpha}_0(\phi)^3 = 91.5298244$ GeV	$(40)(\sqrt{2})\left(\frac{1}{\phi}\right)$ $= (40)(\sqrt{2})(1+\phi)$ $= 91.5298244$ GeV	91.188 GeV
η'	$(m_\eta)\left(\frac{7}{4}\right) = \left[(4)(\overline{\alpha}_{gs})\right]^2\frac{7}{80} = \left[(4)\frac{20\left(\frac{1}{\phi}\right)^2}{2}\right]^2\frac{7}{80} = (140)\left(\frac{1}{\phi}\right)^4$ $= (140)(7-\phi^4) = 959.5742755$ MeV	$7\overline{\alpha}_0 = 959.5742755$ MeV	$\approx (226)\left(\frac{1}{\phi}\right)^3$ $= (226)(4+\phi^3) = 957.351$ MeV	957.5 MeV

Υ_{1s}	$(m_b)\sqrt{5} - 20\left(\frac{m_d}{m_u}\right) = (10)\left(\frac{1}{\phi}\right)^3\sqrt{5} - (20)\left(\frac{1}{\phi}\right)$ $= (10)\left(\frac{1}{\phi}\right)\left(\left(\frac{1}{\phi}\right)^2\sqrt{5} - 2\right) = (10)\left(\frac{1}{\phi}\right)\left(\left(\frac{1}{\phi}\right)^4 - 3\right)$ $= (10)(1+\phi)(4-\phi^4) = 9459.775272$ MeV	$\left(\frac{1}{\phi}\right)\left(\frac{\overline{\alpha}_0}{2} - 30\right) = 9459.775272$ MeV $\simeq (\overline{\alpha}_0)(69) = 9458.66$ MeV	$\approx (853)\left(\frac{1}{\phi}\right)^5$ $= (853)(11+\phi^5)$ $= 9459.149$ MeV	9460.3 Mev
Λ	$(m_{\pi\pm})(8) = \left(\overline{\alpha} + \frac{5}{2}\right)(8) = 8\left(\frac{5}{2}\right)\left(\frac{1}{\phi}\right)^4(8+\phi^4)$ $= (20)\left(\frac{1}{\phi}\right)^4(8+\phi^4) = 20(7-\phi^4)(8+\phi^4) = 1116.6$ MeV	$\overline{\alpha}_0(8+\phi^4) = 1116.6$ MeV	$\approx (426)\left(\frac{1}{\phi}\right)^2$ $= (428)(3-\phi^2)$ $= 1115.2824$ MeV	1115.63 MeV
$J/\Psi_{\rm is}$	$(m_{\pi\pm})(22+k) = \left(\overline{\alpha} + \frac{5}{2}\right)(22+2\phi^5)$ $= 2\left(\frac{5}{2}\right)\left(\frac{1}{\phi}\right)^4(8+\phi^4)\cdot(11+\phi^5) = 5\left(\frac{1}{\phi}\right)^4(8+\phi^4)\left(\frac{1}{\phi}\right)^5$ $= 5\left(\frac{1}{\phi}\right)^9(8+\phi^4) = 3095.9318842$ MeV	$\frac{\overline{\alpha}_0}{4}(8+\phi^4)\ (11+\phi^5)$ $= 3095.931942$ MeV	$\approx (107)\left(\frac{1}{\phi}\right)^7$ $= (107)(29+\phi^7)$ $= 3095.942897$ MeV	3096.9 MeV
ρ (770)	$\langle m_\pi\rangle(5+\phi) = \overline{\alpha}_0(5+\phi)$ $= 20(7-\phi^4)(5+\phi) = 20\left(\frac{1}{\phi}\right)^4\cdot\left(4+\frac{1}{\phi}\right)$ $= 770.1315561$ Mev	$\overline{\alpha}_0(5+\phi) = 770.1315561$ MeV	$\approx (294)\left(\frac{1}{\phi}\right)^2$ $= (294)(3-\phi^2)$ $= 769.7019928$ MeV	770 MeV
ω (783)	$(m_{\pi\pm})(5+\phi) = \left(\overline{\alpha}_0 + \frac{5}{2}\right)(5+\phi)$ $= \frac{5}{2}(7-\phi^4)(8+\phi^4)(5+\phi)$ $= \frac{5}{2}\left(\frac{1}{\phi}\right)^4(8+\phi^4)(5+\phi)$ $= 784.131$ MeV	$\frac{\overline{\alpha}_0}{4}(8+\phi^4)(5+\phi) = 784.131$ MeV $\simeq (\overline{\alpha}_0 - (6+\phi))(6) = 782.7840$ MeV	$\approx (114)\left(\frac{1}{\phi}\right)^4$ $= (114)(7-\phi^4)$ $= 781.3676244$ MeV	782.0 MeV
η_0	$(496-k^2)(6) = 6\overline{\alpha}_0(4-\phi^2) = 6\cdot(20)\left(\frac{1}{\phi}\right)^4(4-\phi^2)$ $= (120)(7-\phi^4)(4-\phi^2) = 2974.917961$ MeV	$6\overline{\alpha}_0(4-\phi^2) = 6\overline{\alpha}_0(3+\phi)$ $= 2977.917961$ MeV $\simeq (\overline{\alpha}_0 - 1/\phi)(22)$ $= 2988.208117$ MeV	$\approx (1138)\left(\frac{1}{\phi}\right)^2$ $= (1138)(3-\phi^2)$ $= 2979.322679$ MeV	2979.6 MeV

For our expressions we need the relation between ϕ^n and $1/\phi^n$. The powers of ϕ are known. These are

$$\phi = \frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}+(-1)\cdot 1}{2} = \frac{a_1\sqrt{5}+(-1)b_1}{2}$$

$$\phi^2 = \frac{-\sqrt{5}+3}{2} = \frac{(+)\sqrt{5}+3}{2} = \frac{(-1)(a_1+a_2)\sqrt{5}+(b_1+b_2)}{2}$$

$$\phi^2 = \frac{2\sqrt{5}-4}{2} = \frac{2\sqrt{5}+(-1)\cdot 4}{2}$$

$$\phi^4 = \frac{-3\sqrt{5}+7}{2} = \frac{(-1)\cdot 3\sqrt{5}+7}{2}$$

$$\vdots$$

$$\phi^n = \frac{(-1)^{n-1}a_n\sqrt{5}+(-1)^n b_n}{2}, \quad n>2,\ n\in\ N$$

The members $a_1 = 1,\ a_2 = 1,\ a_3 = 2,\ a_4 = 3,\ldots,\ a_n = a_{n-2} + a_{n-1}$ form the Fibonacci series. The members $b_1 = 1$, $b_2 = 3$, $b_3 = 4,\ldots,$ form another series $b_n = b_{n-2} + b_{n-1}$.

The relation between ϕ^n and $1/\phi^n$ is easily written.

We write the expression for $1/\phi, 1/\phi^2,\ldots,1/\phi^n$, as following

$$\frac{1}{\phi} = 1+\phi$$

$$\frac{1}{\phi^2} = 3-\phi^2 = 3+(-1)\phi^2$$

$$\frac{1}{\phi^3} = 4+\phi^3$$

$$\frac{1}{\phi^4} = 7-\phi^4 = 7+(-1)\phi^4$$

$$\frac{1}{\phi^5} = 11+\phi^5$$

$$\vdots$$

$$\frac{1}{\phi^n} = (b_{n-2}+b_{n-1}) + (-1)^{b-1}\phi^n, \quad n>2,\ n\in N$$

With this generalization it is possible to express the theoretical masses of particles in terms of ϕ and $1/\phi$ and the nearest value to the experimental value may be found in powers of ϕ and $1/\phi$.

2. Numerical results

The results of our calculations of the mass spectrum of the standard model are in Table 1 which includes 39 particles.

3. Conclusion

Theoretical masses of the elementary particles of the standard model are expressed in terms of ϕ and $1/\phi$ in the most simple form. The experimental values of masses of the particles are also expressed in powers of ϕ and $1/\phi$, which gives the nearest value to the experimentally known value. Both values were found to be in more than excellent agreement.

A final note is regarding the appearance of the units (M.e.V.) in what is a pure dimensionless number such as $\bar{\alpha}_0 \cong 137$. For instance $m_{\pi^\pm} = \bar{\alpha}_0 + \left(\frac{5}{2}\right) = 139.57$ MeV and $m_N = \frac{(\bar{\alpha}_0)^2}{20}$ MeV. This can be explained easily in several different ways. First one can give a mechanical model such as that of Sidharth [5] where the π meson are considered to be made of an electron and a positron rotating around the centre of the mass. That way Sidharth found (see page 49 of Ref. [5]) that

$$m_{\pi^{+,-,0}} = \frac{\hbar c}{e^2} = \frac{1}{\alpha_0} = \bar{\alpha}_0 \cong 137\ (\mathrm{MeV}/c^2)$$

In the case of $\mathscr{E}^{(\infty)}$ theory we can see every particle as a scaling of another particle. Let us start from the electron–positron mass like Sidharth and scale the mass of that of $\pi^{\pm}$ meson, so that

$$m_{\pi^{\pm}} = (\lambda_{\pi^{\pm}})(m_{e^{\pm}})$$

In this case we have

$$m_{e^{\pm}} = 0.511 \text{ MeV}$$

and

$$\lambda_{\pi^{\pm}} = (2\bar{\alpha}_0 - 1) = 273.16$$

where

$$\bar{\alpha}_0 = (20)\left(\frac{1}{\phi}\right)^4 = 137.082$$

Consequently

$$m_{\pi^{\pm}} = (2\bar{\alpha}_0 - 1)(m_{e^{\pm}}) \cong 139.57 \text{ MeV}$$

which is almost exactly what is found experimentally.

For the neutral pion π^0, we have something similar, namely

$$m_{\pi^0} = (\lambda_{\pi^0})(m_e)$$

where this time λ_{π^0} is given by

$$\lambda_{\pi^0} = (2\bar{\alpha}_0 - 10)$$

so that

$$m_{\pi^0} = (2\bar{\alpha}_0 - 10)(m_e) \cong 134.98 \text{ MeV}$$

Again this is exactly as the value found experimentally.

One should note that the scaling "exponents" are made up entirely from a combinatoric of the permissible dimensions of $\mathscr{E}^{(\infty)}$ theory according to the dimensional function of the fusion algebra with the fundamental elements 1 and ϕ.

Next we consider the neutron as a scaled expectation meson. That is to say as a scaling of the totally unstable hypothetical meson $\langle m_\pi \rangle = 137.082\ldots$MeV, form which the $\pi^{\pm}$ and π^0 mesons are created and for which the mass is estimated to be

$$\langle m_\pi \rangle \cong \sqrt{(m_{\pi^{\pm}})(m_{\pi^0})(\text{MeV})^2} \cong 137 \cong \bar{\alpha}_0 \text{ MeV}$$

That way one finds

$$m_N = (\lambda_N)(\langle m_\pi \rangle)$$

with

$$\lambda_N = \left(\frac{\bar{\alpha}_0}{(2)(10)}\right)$$

to be

$$m_N = \left(\frac{\bar{\alpha}_0}{20}\right)(\langle m_\pi \rangle) \cong 939.58 \text{ MeV}$$

This agrees completely with the experimentally found mass. However, since numerically both $\langle m_\pi \rangle$ and $(\lambda_N)(20)$ are given by

$$\langle m_\pi \rangle = (\lambda_N)(20) = \bar{\alpha}_0.$$

One can simply write

$$m_N = \frac{(\bar{\alpha}_0)^2}{20} \text{ MeV}$$

which explains the fusion of the MeV units with the pure number of for instance the inverse of the fine structure constant of Sommerfield α_0.

Acknowledgement

The author would like to thank Prof. M.S. El Naschie, formerly from University of Cambridge, England, for advice and detailed comments.

References

[1] El Naschie MS. On the exact mass spectrum of quarks. Chaos, Solitons and Fractals 2002;14:36–376.
[2] El Naschie MS. On a class of general theories for high energy particle physics. Chaos, Solitons and Fractals 2002;14:649–68.
[3] El Naschie MS. Heterotic string space time from probability theory. Chaos, Solitons and Fractals 2001;12:617–21.
[4] El Naschie MS. Four as the expectation value of the set of all positive integer and the geometry of four manifolds. Chaos, Solitons and Fractals 1998;9:1625–2629.
[5] Sidhart BG. Chaotic Universe, Nova, New York: 2001.